Class XI Session 2025-26
Subject - Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. $\tan 20^o + \tan 40^o + \sqrt{3} \tan 20^o \tan 40^o$ is equal to **[1]**

a) $\frac{\sqrt{3}}{2}$ b) 1

c) $\sqrt{3}$ d) $\frac{\sqrt{3}}{4}$

2. The domain of the function $f(x)=\sqrt{x-1}+\sqrt{6-x}$ **[1]**

a) [1, 6] b) [2, 6]

c) $(-\infty, 6)$ d) [-2, 6]

3. The marks obtained by 13 students in a test are 10, 3, 10, 12, 9, 7, 9, 6, 7, 10, 8, 6 and 7. The median of this data is: **[1]**

a) 9 b) 8

c) 10 d) 7

4. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$, then $\frac{dy}{dx} =$ **[1]**

a) y + 1 b) y^2

c) y d) y - 1

5. The angle between the lines 2x - y + 3 = 0 and x + 2y + 3 = 0 is **[1]**

a) 45°

b) 60°

c) 90°

d) 30°

6. What is the perpendicular distance of the point P (6, 7, 8) from xy-plane? **[1]**

a) 5

b) 7

c) 6

d) 8

7. Mark the correct answer for $\frac{(3-5i)}{(-2+3i)}$ = ? **[1]**

a) $\left(\frac{21}{13}-\frac{3}{13}i\right)$

b) $\left(\frac{21}{13}-\frac{1}{13}i\right)$

c) $\left(\frac{-21}{13}+\frac{1}{13}i\right)$

d) $\left(\frac{21}{13}+\frac{1}{13}i\right)$

8. The number of different four digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once is **[1]**

a) 120

b) 24

c) 100

d) 96

9. $\lim\limits_{x\to 0}\frac{x}{\tan x}=$ **[1]**

a) 2

b) 4

c) 1

d) 0

10. If $\tan\theta=\frac{1}{2}$ and $\tan\phi=\frac{1}{3}$, then value of $\theta+\phi$ is **[1]**

a) $\frac{\pi}{6}$

b) $\frac{\pi}{4}$

c) 0

d) π

11. If a set A has n elements then the total number of subsets of A is **[1]**

a) 2^n

b) 2n

c) n

d) n^2

12. The 14th term from the end in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$ is **[1]**

a) ${}^{17}C_6(\sqrt{x})^{11}\cdot y^3$

b) ${}^{14}C_2x^{11/3}\cdot y^2$

c) $-{}^{17}C_5x^6(\sqrt{y})^5$

d) ${}^{17}C_4x^{13/2}\cdot y^2$

13. The coefficient of x^2 in the expansion of $\left(3x-\frac{1}{x}\right)^6$ is **[1]**

a) 3645

b) 405

c) 1215

d) 2430

14. If x and a are real numbers such that a > 0 and |x| > a, then **[1]**

a) $x\in[-\infty,a]$

b) x ∈ (-a, a)

c) $x\in(-a,\infty)$

d) $x\in(-\infty,-a)\cup(a,\infty)$

15. For any set A, (A')' is equal to **[1]**

a) A

b) A'

c) A"

d) ϕ

16. If R is a relation on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} given by x R y ⇔ y = 3x, then R = **[1]**

a) {(3, 1), (6, 2), (9, 3)}

b) none of these

c) {(3, 1), (2, 6), (3, 9)}

d) {(3, 1), (6, 2), (8, 2), (9, 3)}

17. Let f(x) = x^2 then, dom (f) and range (f) are respectively **[1]**

a) R and R - {0}

b) R and R

c) R^+ and R^+

d) R and R^+

18. There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip? **[1]**

a) 144

b) 72

c) 14

d) 19

19. **Assertion (A):** A = {(1, 5), (1, 5), (7, -8), (7, -8), (7,-8)} is function. **[1]**

Reason (R): A function is a relation which describes that there should be only one output for each input (or), we can say that a special kind of relation (a set of ordered pairs), which follows a rule i.e., every x-value should be associated with only one y-value is called a function.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The sum of infinite terms of a geometric progression is given by $S_\infty = \frac{a}{1-r}$, provided |r| < 1. **[1]**

Reason (R): The sum of n terms of Geometric progression is $S_n = \frac{a(r^n-1)}{r-1}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Draw the graph of the exponential function: f(x) = 2^x. **[2]**

OR

Find the domain and range of the real function $f(x) = \sqrt{9 - x^2}$.

22. Differentiate the function with respect to x: $\frac{x^2-x+1}{x^2+x+1}$ **[2]**

23. A bag contains 4 white and 5 black balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is white. **[2]**

OR

Find the probability that in a random arrangement of the letters of the word 'UNIVERSITY', the two I's do not come together.

24. Prove that $(A \cap B')' \cup (B \cap C) = A' \cup B$. **[2]**

25. Find the image of: (-4, 0, 0) in the xy-plane. **[2]**

Section C

26. If A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}, then verify that **[3]**

i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

ii. $A \times C$ is a subset of $B \times D$.

27. Find the equation of a straight line parallel to 2x + 3y + 11 = 0 and which is such that the sum of its intercepts on the axes is 15. **[3]**

28. Show that the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$ does not contain any term involving x^9. **[3]**

OR

Find the coefficient of x^6 in the expansion $\left(x - \frac{1}{x^2}\right)^{24}$

29. Find $\lim\limits_{x \to 1} f(x)$ where f(x) = $\begin{cases} x^2 - 1, & x \le 0 \\ -x^2 - 1, & x > 1 \end{cases}$ **[3]**

OR

Evaluate : $\lim\limits_{x \to 1} \frac{\sqrt{x^2-1}+\sqrt{x-1}}{\sqrt{x^2-1}}$, x > 1.

30. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term. **[3]**

OR

Sum the series .4 + .44 + .444 + ...to n terms.

31. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50th word? **[3]**

Section D

32. The mean and standard deviation of 20 observation are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases: **[5]**

(i) If wrong item is omitted.

(ii) If it is replaced by 12

33. Find the equation of the hyperbola with vertices at $(\pm 6, 0)$ and foci at $(\pm 8, 0)$. **[5]**

OR

Find the equation of the ellipse, whose foci are(± 3, 0) and passing through (4, 1).

34. Solve the following system of linear inequalities **[5]**

$\frac{4x}{3} - \frac{9}{4} <$ x + $\frac{3}{4}$ and $\frac{7x-1}{3}$ - $\frac{7x+2}{6}$ > x.

35. Prove that: $\cos^3 x \sin 3x + \sin^3 x \cos 3x = \frac{3}{4} \sin 4x$. **[5]**

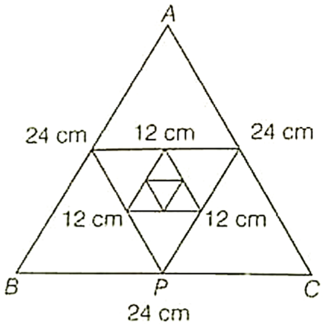
OR

Prove that: $\sin 6^o \sin 42^o \sin 66^o \sin 78^o = \frac{1}{16}$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



i. Find the sum of perimeter of all triangles (in cm)? (1)

ii. Find the sum of area of all the triangle (in sq cm)? (1)

iii. Find the sum of perimeter of first 6 triangle is (in cm)? (2)

OR

iv. Find the sum of areas of first 4 triangles in sqcm? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

In a hostel 60% of the students read Hindi newspapers, 40% read English newspapers and 20% read both Hindi and English newspapers.

1. A student is selected at random. She reads Hindi or English newspaper? (1)
2. A student is selected at random. Did she read neither Hindi nor English newspapers? (1)
3. A student is selected at random. She reads Hindi but not English Newspaper? (2)

OR

A student is selected at random. She reads English but not Hindi Newspaper? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

In a library 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.

i. Find the number of students who read none of the subject. (1)

ii. Find the number of students who read atleast one of the subject. (1)

iii. Find the number of students who read only one of the subjects. (2)

OR

Find the number of students who read only mathematics. (2)

Solution

Section A

1.

(c) $\sqrt{3}$

Explanation:

tan 20^o + tan 40^o + $\sqrt{3}$ tan 20^o tan 40^o

= tan 60^o (1 - tan 20^o tan 40^o) + tan 60^o tan 20^o tan 40^o [Using tan $60^o = \frac{\tan 20+\tan 40}{1-\tan 20 \tan 40}$ and tan $60^o = \sqrt{3}$]

= tan 60^o - tan 60^o tan 20^o tan 40^o + tan 60^o tan20^o tan 40^o

= tan 60^o

$= \sqrt{3}$

2. **(a)** [1, 6]

Explanation:

For f(x) to be real, we must have,

$x - 1 \geqslant 0$ *and* $6 - x \geqslant 0$

$\Rightarrow\ x \geqslant \varphi$ *and* $x - 6\ \leqslant 6$

$\therefore$ Domain = [1, 6]

3.

(b) 8

Explanation:

Arrange the given data in ascending order, we get

3, 6, 6, 7, 7, 7, 8, 9, 9, 10, 10, 10, 12

Total terms, n = 13 (odd)

$\therefore$ Median = $\left(\frac{n+1}{2}\right)$th term

= $\left(\frac{13+1}{2}\right)$th term = 7th term = 8

4.

(c) y

Explanation:

y = 1 + $\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$ + ...

Differentiating both sides with respect to x, we get $\frac{dy}{dx} = \frac{d}{dx}\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^2}{3!} + \dots\right)$

= $\frac{d}{dx}(1) + \frac{d}{dw}\left(\frac{x}{11}\right) + \frac{d}{dw}\left(\frac{x^2}{2!}\right) + \frac{d}{dw}\left(\frac{x^3}{3!}\right) + \frac{d}{dx}\left(\frac{x^4}{4!}\right)$ + ...

= $\frac{d}{dx}(1) + \frac{1}{1!}\frac{d}{dx}(x) + \frac{1}{2!}\frac{d}{dw}\left(x^2\right) + \frac{1}{3!}\frac{d}{dw}\left(x^3\right) + \frac{1}{4!}\frac{d}{dw}\left(x^4\right)$ + ...

= 0 + $\frac{1}{1!}\times$ 1 + $\frac{1}{2!} \times 2\alpha + \frac{1}{3!} \times 3\alpha^2 + \frac{1}{4!} \times 4\alpha^3$ + ... (y = $\alpha^2 \Rightarrow \frac{dy}{\partial\alpha}$ = nα^{n-1})

= 1 + $\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$ + ... $\left[\frac{x}{n!} = \frac{1}{(n-1)!}\right]$

= y

$\therefore \frac{dy}{dx}$ = y

5.

(c) 90°

Explanation:

Let m_1 and m_2 be the slope of the lines 2x - y + 3 = 0 and x + 2y + 3 = 0, respectively.

Let θ be the angle between them.

Here,

$m_1 = 2$ and $m_2 = -\frac{1}{2}$

$\therefore m_1 m_2 = -1$

Therefore, the angle between the given lines is 90° , as it satisfy the condition of product of slopes of two lines is -1.

6.

(d) 8

Explanation:

Let L be the foot of the perpendicular drawn from the point P (6, 7, 8) to the XY-plane and the distance of this foot L from P is z-coordinate of P = 8 units.

7.

(c) $\left(\frac{-21}{13}+\frac{1}{13}i\right)$

Explanation:

$\frac{(3-5i)}{(-2+3i)} = \frac{(3-5i)}{(-2+3i)} \times \frac{(-2-3i)}{(-2-3i)} = \frac{(3-5i)(-2-3i)}{(-2)^2-(3i)^2}$

$= \frac{-6-9i+10i+15i^2}{(4-9i^2)} = \frac{-6+i-15}{(4+9)} = \frac{(-21+i)}{13} = \left(\frac{-21}{13}+\frac{1}{13}i\right)$

8.

(b) 24

Explanation:

Four-digit numbers are to be formed from the digits 2, 3, 4, 7 without repetition

Therefore, the required 4-digit numbers = ${}^4P_4 = 4! = 4 \times 3 \times 2 \times 1 = 24$.

9.

(c) 1

Explanation:

$\lim\limits_{x\to 0} \frac{x}{\tan x}$

$= \lim\limits_{x\to 0} \frac{1}{\frac{\tan x}{x}}$

$= \frac{1}{1}$

= 1

10.

(b) $\frac{\pi}{4}$

Explanation:

It is given that $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

Now,

$\tan(\theta+\phi) = \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$

$= \frac{\frac{5}{6}}{\frac{5}{6}}$

= 1

$\therefore \theta+\phi = \frac{\pi}{4}\left(\tan\frac{\pi}{4} = 1\right)$

11. **(a)** 2^n

Explanation:

The total no of subsets = 2^n

12.

(d) $^{17}C_4 x^{13/2} \cdot y^2$

Explanation:

Here,it is given expansion is $(\sqrt{x} - \sqrt{y})^{17}$

pth term from the end = (n - p + 2)th term.

14th term from the end = (17 - 14 + 2)th term = 5th term.

$T_{r+1} = (-1)^r . {}^{17}C_r \cdot (\sqrt{x})^{(17-r)} \cdot (\sqrt{y})^r$

$\Rightarrow T^5 = T_{4+1} = (-1)^4 . {}^{17}C_4 \cdot (\sqrt{x})^{(17-4)} \cdot (\sqrt{y})^4 = {}^{17}C_4 x^{13/2} \cdot y^2$

13.

(c) 1215

Explanation:

Here,it is given general term in the expansion $\left(3x - \frac{1}{x}\right)^6$ is

$T_{r+1} = (-1)^r . {}^6C_r (3x)^{(6-r)} \cdot \left(\frac{1}{x}\right)^r = (-1)^r . {}^6C_r . 3^{(6-r)} x^{(6-2r)}$

Putting 6 - 2r = 2, we get 2r = 4 ⇒ r = 2.

$\therefore T_3 = (-1)^2 . {}^6C_2 . 3^{(6-2)} . x^2 = {}^6C_2 \times 3^4 = \frac{6\times5}{2\times1} \times 8! = 1215.$

14.

(d) $x \in (-\infty, -a) \cup (a, \infty)$

Explanation:

|x| > a

⇒ x < -a or x > a

⇒x ∈ (−∞, -a) ∪ (a, ∞)

15. **(a)** A

Explanation:

We have to find (A')' = ?

Now, A = U\A

⇒ (A')' = (U\A)' = U'\A'

⇒ (A')' = U'\(U\A)

⇒ (A')' = U'\(U\A)

⇒ (A')' = A

16.

(b) none of these

Explanation:

∴ For A = {1, 2, 3, 4, 5, 6, 7, 8, 9} the satisfying complete relation is:

R = {(1, 3), (2, 6), (3, 9)}

17.

(d) R and R^+

Explanation:

Domain of f = R

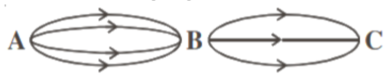
and Range of f = R^+ U {0}

18.

(b) 72

Explanation:

In the following figure:



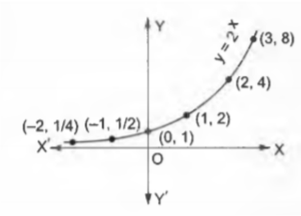
there are 4 bus routes from A to B and 3 routes from B to C. Thus, there are $4 \times 3 = 12$ ways to go from A to C. It is round trip thus, the man will travel back from C to A via B. It is restricted that man can not use same bus routes from C to B and B to A more than once. Therefore, there are $2 \times 3 = 6$ routes for return journey. Thus, the required number of ways = $12 \times 6 = 72$

19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Here, point (1, 5) is repeated twice and (7, -8) is written thrice. We can rewrite it by writing a single copy of thrice. We can rewrite it by writing a single copy of the repeated ordered pairs. So, 'A' is a function.

20.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
Both A and R are true but R is not the correct explanation of A.

Section B

21. Let, $f(x) = 2^X$



$f(x) = 2^X$

Some of the points on the graph are (0 ,1), (1 ,2), (2 ,4), (3 ,8), $\left(-1, \frac{1}{2}\right), \left(-2, \frac{1}{4}\right), \left(-3, \frac{1}{8}\right)$, etc.

And so the graph takes the form, shown in the adjoining figure.

It may be observed here that the given function is strictly increasing.

Also, as the value of x decreases, the corresponding value of the function decreases, and therefore, on the left-hand side of the y-axis, the curve comes closer and closer to the x-axis.

This is the case of the exponential function a^X where a > 1 required graph of the function is shown in fig.

OR

It is clear that, $f(x) = \sqrt{9 - x^2}$ is not defined when $(9 - x^2) < 0$, i.e.

When $x^2 > 9$ i,.e when x > 3 or x < -3

dom (f) = $|x \in R : -3 \leq x \leq 3|$

Also, y = $\sqrt{9 - x^2} \Rightarrow y^2 = \left(9 - x^2\right)$

$\Rightarrow x = \sqrt{9 - y^2}$

clearly, x is not defined when $\left(9 - y^2\right) < 0$

but $\left(9 - y^2\right) < 0 \Rightarrow y^2 > 9$

$\Rightarrow y > 3$ or $y < -3$

range $(f) = \{y \in R : -3 \leq y \leq 3\}$.

22. Using quotient rule, we have

$$\frac{d}{dx}\frac{x^2-x+1}{x^2+x+1}$$

$$= \frac{\left(x^2+x+1\right)\frac{d}{dx}\left(x^2-x+1\right)-\left(x^2-x+1\right)\frac{d}{dx}\left(x^2+x+1\right)}{\left(x^2+x+1\right)^2}$$

$$= \frac{\left(x^2+x+1\right)(2x-1)-\left(x^2-x+1\right)(2x+1)}{\left(x^2+x+1\right)^2}$$

$$= \frac{\left(x^2+1-x\right)(2x-1)-\left(x^2-x+1\right)(2x+1)}{\left(x^2+x+1\right)^2}$$

$$= \frac{2x^3+2x+2x^2-x^2-1-x-2x^3+2x^2-2x-x^2+x-1}{\left(x^2+x+1\right)^2}$$

$$= \frac{2x^2-2}{\left(x^2+x+1\right)^2}$$

$= \frac{2(x^2-1)}{(x^2+x+1)^2}$

$\therefore \frac{d}{dx}\frac{x^2-x+1}{x^2+x+1} = \frac{2(x^2-1)}{(x^2+x+1)^2}$

23. We know that,

Probability of occurrence of an event = $\frac{\text{Total no.of Desired outcomes}}{\text{Total no.of outcomes}}$

By permutation and combination, total no.of ways to pick r objects from given n objects is ^nC_r

Now, total no.of ways to pick a ball from 9 balls is $^9c_1 = 9$

Our desired output is to pick a white ball. So, no.of ways to pick a white ball from 4 white balls (because the white ball can be picked from only white balls) is $^4c_1 = 4$

Therefore, the probability of picking a white ball = $\frac{4}{9}$

Conclusion: Probability of picking a white ball from 4 white balls and 5 white balls is $\frac{4}{9}$.

OR

According to the question, we can write,

Given: word "UNIVERSITY"

Formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability that two I's do not come together

total possible outcomes for arrangement of alphabets are 10!

therefore n(S)= 10!

let E be the event that both I's come together

n(E)= 2! × 9!

probability of occurrence of this event is

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{2!9!}{10!} = \frac{2}{10} = \frac{1}{5}$

Let E' be the event that both I's do not come together

Therefore, the probability that two I's do not come together is

P(E') = 1 - P(E)

$P(E') = 1 - \frac{1}{5} = \frac{4}{5}$

24. To Prove: $(A \cap B')' \cup (B \cap C) = A' \cup B$

LHS = $(A \cap B')' \cup (B \cap C)$

$= (A' \cup (B')') \cup (B \cap C)$ [According to DeMorgan's Law]

$= (A' \cup B) \cup (B \cap C)$

$= ((A' \cup B) \cup B) \cap ((A' \cup B) \cup C)$

$= (A' \cup (B \cup B)) \cap (A' \cup B \cup C)$

$= (A' \cup B) \cap (A' \cup B \cup C)$

$= (A' \cup B)$ **= RHS**

Hence Proved.

25. Given: Point is (-4, 0, 0)

Since we need to find its image in xy-plane, sign of its z-coordinate will change

So, Image of point (-4, 0, 0) is (-4, 0, 0)

Section C

26. i. We have, $A = \{1,2\}$, $B = \{1,2,3,4\}$, $C = \{5,6\}$ and $D = \{5,6,7,8\}$

$\therefore (B \cap C) = \{1,2,3,4\} \cap \{5,6\} = \phi$

and $A \times (B \cap C) = \{1,2\} \times \phi = \phi$ .... (i)

Now, $A \times B = \{1, 2\} \times \{1, 2, 3, 4\}$

= $\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)\}$

and $A \times C = \{1,2\} \times \{5,6\}$

= $\{(1,5), (1,6), (2,5), (2,6)\}$

$\therefore \quad (A \times B) \cap (A \times C) = \phi$ .... (ii)

From Eqs. (i) and (ii),

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

ii. Now, $A \times C = \{1, 2\} \times \{5, 6\}$
= $\{(1,5), (1,6), (2,5), (2,6)\}$ ......(iii)
and $B \times D$ ={1, 2, 3, 4} × {5, 6, 7, 8}
= $\{(1,5), (1,6), (1,7), (1,8), (2,5), (2,6), (2,7), (2,8), (3,5), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8)\}$ ..... (iv)
From Eqs. (iii) and (iv), we can say,
$A \times C$ is a subset of $B \times D$.

27. Equation of a line parallel to 2x + 3y + 11 = 0 is
2x + 3y + λ = 0, where λ is a constant ...(i)
To find x-intercept of this line, we put y = 0 in its equation.
Put, y = 0 in equation (i), we obtain,
$\Rightarrow$ 2x + λ, = 0
$\Rightarrow$ x = $-\frac{\lambda}{2}$
So, x-intercept = $-\frac{\lambda}{2}$
To find y-intercept of this line, we put x = 0 in its equation.
Put, x = 0 in (i), we obtain,
3y + λ = 0
$\Rightarrow$ y = $-\frac{\lambda}{3}$
So, y-intercept = $-\frac{\lambda}{3}$
It is given that the sum of the intercepts of the line (i) on the coordinate axes is 15
$\therefore \left(-\frac{\lambda}{2}\right) + \left(-\frac{\lambda}{3}\right)$ = 15
$\Rightarrow -\frac{5\lambda}{6}$ = 15
$\Rightarrow \lambda$ = -18
Put, λ = -1 8 in (i), we obtain,
2x + 3y -1 8 = 0
Hence, the equation of the required line is 2x + 3y - 18 = 0

28. For the given function. $\left(2x^2 - \frac{1}{x}\right)^{20}$
We have ,a = $2x^2$, $b = \frac{-1}{x}$ and n = 20
Now using a formula,
$t_{r+1} = \binom{n}{r} a^{n-r} b^r$
$= \binom{20}{r} \left(3x^2\right)^{20-r} \left(\frac{-1}{x}\right)^r$
$= \binom{20}{r} (3)^{20-r} \left(x^2\right)^{20-r} (-1)^r (x)^{-r}$
$= \binom{20}{r} (3)^{20-r} (x)^{40-2r} (-1)^r (x)^{-r}$
$= \binom{20}{r} (3)^{20-r} (x)^{40-2r-r} (-1)$
To get coefficient of x^9 we must have,
$(x)^{40-3r} = (x)^9$
40 - 3r = 9
3r = 31
r = 10.3333
As $\binom{20}{r} = \binom{20}{10.3333}$ is not possible.
Therefore, the term containing x^9 does not exist in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$.

OR

Comparing $\left(x - \frac{1}{x^2}\right)^{24}$ with $(a + b)^n$, we have
a = x, $b = \left(\frac{1}{x^2}\right)$ and n = 24
We know that $T_{r+1} = {}^nC_r a^{n-r} b^r$
$\therefore T_{r+1} = {}^{24}C_r x^{24-r} \cdot \left(-\frac{1}{x^2}\right)^r$
$= {}^{24}C_r (-1)^r x^{24-3r}$
Now 24 - 3r = 6
$\Rightarrow 3r = 24 - 6 \Rightarrow r = 6$

$\therefore$ Coefficient of $x^6 = {}^{24}C_6(-1)^6$

$= {}^{24}C_6 = \frac{24!}{18!6!} = 134596$

29. Here $\lim_{x\to 1} f(x) = \begin{cases} x^2-1, & x\le 0 \\ -x^2-1, & x>1 \end{cases}$

L.H.L. $\lim_{x\to 1} f(x) = \lim_{x\to 1^-}(x^2-1)$

Put x = 1 - h as $x\to 1,\ h\to 0$

$\therefore\ \lim_{h\to 0}[(1+h)^2-1] = \lim_{h\to 0}[1+h^2-1]$

R.H.L. $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(-x^2-1)$

Put x = 1 + h as $x\to 1,\ h\to 0$

$\therefore\ \lim_{h\to 0}[-(1+h)^2-1] = \lim_{h\to 0}[-1-h^2-2h-1]$

$= -(0)^2 - 2\times 0 - 2 = -2$

$\therefore$ LHL $\neq$ RHL

Therefore limit of given function does not exist

OR

Given: $\lim_{x\to 1}\frac{\sqrt{x^2-1}+\sqrt{x-1}}{\sqrt{x^2-1}}$

$= \lim_{x\to 1}\frac{(\sqrt{x^2-1}+\sqrt{x-1})}{\sqrt{x^2-1}} \times \frac{(\sqrt{x^2-1}-\sqrt{x-1})}{(\sqrt{x^2-1}-\sqrt{x-1})} \times \frac{\sqrt{x^2-1}}{\sqrt{x^2-1}}$

$= \lim_{x\to 1}\frac{[(x^2-1)-(x-1)]\times\sqrt{x^2-1}}{(x^2-1)(\sqrt{x^2-1}-\sqrt{x-1})}$

$= \lim_{x\to 1}\frac{(x^2-x)\sqrt{x^2-1}}{(x^2-1)(\sqrt{x^2-1}-\sqrt{x-1})}$

$= \lim_{x\to 1}\frac{x(x-1)\sqrt{x^2-1}}{(x-1)(x+1)(\sqrt{x^2-1}-\sqrt{x-1})}$

$\lim_{x\to 1}\frac{x(\sqrt{x-1})(\sqrt{x+1})}{(x+1)(\sqrt{x-1})(\sqrt{x+1}-1)}$

$= \frac{\sqrt{2}}{2(\sqrt{2}-1)}$

$= \frac{\sqrt{2}}{2\times(\sqrt{2}-1)} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{\sqrt{2}+1}{\sqrt{2}}$

30. Let a be the first term and r be the common ratio of given G.P.

Given: a + ar = -4

$\Rightarrow$ a(1 + r) = -4 ……..(i)

And $a_5 = 4a_3$

$\Rightarrow ar^4 = 4ar^2$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

Putting r = 2 $r = 2$ in eq. (i), we get a(1 + 2) = -4

$\Rightarrow a = \frac{-4}{3}$

Therefore, required G.P. is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$

Putting r = -2 in eq. (i), we get a(1 - 2) = -4

$\Rightarrow$ a = 4

Therefore, required G.P. is 4, -8, 16, -32,.......

OR

Given series

.4 + .44 + .444 +... to n terms

= 4 $\times$ {.1 + .11 + .111 +... to n terms}

$= \frac{4}{9} \times$ {.9 + .99 + .999 +... to n terms}

$= \frac{4}{9} \times$ {(1 - .1) + (1 - .01) + (1 - .001) +... to n terms)

$= \frac{4}{9} \times$ {(1 +1 +... to n terms) - (.1 + .01 + .001 +... to n terms)}

$= \frac{4}{9} \times \left[n - \frac{1\times\{1-(.1)^n\}}{(1-.1)}\right]$ $\{\because S_n = \frac{a(1-r^n)}{(1-r)}\}$

$= \frac{4}{9} \times \left[n - \frac{\frac{1}{10} \cdot \left\{ 1 - \frac{1}{(10)^n} \right\}}{\left(1 - \frac{1}{10}\right)} \right]$

$= \frac{4}{9} \times \left\{ n - \frac{(10^n - 1)}{9 \cdot 10^n} \right\} = \frac{4}{9} \times \left[n - \frac{1}{9} \left\{ 1 - \frac{1}{10^n} \right] \right.$

$= \frac{4}{81} \times \left[9n - \left\{ 1 - \frac{1}{10^n} \right\} \right] = \frac{4}{81} \times \left\{ 9n - 1 + \frac{1}{10^n} \right\}$

Therefore, the required sum is $\frac{4}{81} \times \left(9n - 1 + \frac{1}{10^n} \right)$

31. We have, There are 5 letters in the word AGAIN, in which A appears 2 times. Thus,

the required number of words = $\frac{5!}{2!}$ = 60

To get the number of words starting with A, we fix the letter A at the extreme left position, we then rearrange the remaining 4 letters taken all at a time. There will be as many arrangements of these 4 letters taken 4 at a time as there are permutations of 4 different things taken 4 at a time. Therefore, the number of words starting with A = 4! = 24.

Then, starting with G, the number of words = $\frac{4!}{2!}$ = 12 as after placing G

at the extreme left position, we are left with the letters A, A, I and N. Similarly, there are 12 words starting with the next letter I.

Therefore,total number of words so far obtained = 24 + 12 + 12 = 48

Therefore, the 49th word is NAAGI and he 50th word is NAAIG.

Section D

32. Here n = 20, $\bar{x} = 10$ and $\sigma = 2$

$\therefore\ \bar{x} = \frac{1}{n} \Sigma x_i \Rightarrow n \times \bar{x} = \Sigma x_i$

$\Rightarrow \Sigma x_i = 20 \times 10 = 200$

$\therefore$ Incorrect $\Sigma x_i = 200$

Now $\frac{1}{n} \Sigma x_i^2 - (\bar{x})^2 = \sigma^2$

$\Rightarrow \frac{1}{20} \Sigma x_i^2 - (10)^2 = 4 \Rightarrow \Sigma x_i^2 = 2080$

(i) If wrong item is omitted.

When wrong item 8 is omitted from the data then we have 19 observations.

$\therefore$ Correct Σx_i = Incorrect $\Sigma x_i - 8$

Correct $\Sigma x_i = 200 - 8 = 192$

$\therefore$ Correct mean = $\frac{192}{19}$ = 10.1

Also correct Σx_i^2 = Incorrect $\Sigma x_i^2 - (8)^2$

$\Rightarrow$ Correct $\Sigma x_i^2 = 2080 - 64$ = 2016

$\therefore$ Correct variance $= \frac{1}{19} \left(correct\ \Sigma x_i^2 \right)$ - (correct mean)^2

$= \frac{1}{19} \times 2016 - \left(\frac{192}{19} \right)^2$

$= \frac{2016}{19} - \frac{36864}{361} = \frac{38304 - 36884}{361} = \frac{1440}{361}$

Correct S.D. $= \sqrt{\frac{1440}{361}} = \sqrt{3.99}$ = 1.997

(ii) If it is replaced by 12

When wrong item 8 is replaced by 12

$\therefore$ Correct Σx_i = Incorrect Σx_i - 8 + 12

= 200 - 8 + 12 = 204

$\therefore$ Correct mean = $\frac{204}{20}$ = 10.2

Also correct Σx_i^2 = Incorrect Σx_i^2 - $(8)^2 + (12)^2$

= 2080 - 64 + 144 = 2160

$\therefore$ Correct variance $= \frac{1}{20} (correct\ \Sigma x_1^2)$ - (correct mean)^2

$= \frac{2160}{20} - \left(\frac{204}{20} \right)^2$

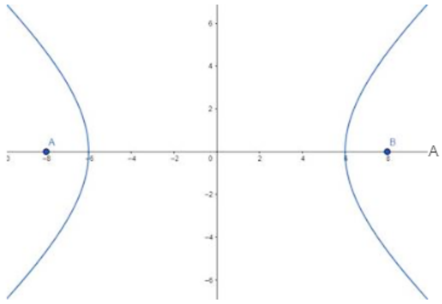
$= \frac{2160}{20} - \frac{41616}{400} = \frac{43200 - 41616}{400} = \frac{1584}{400}$

Correct S.D. $= \sqrt{\frac{1584}{400}} = \sqrt{3.96}$ = 1.989

33. Given: hyperbola with vertices at ($\pm$6, 0) and foci at ($\pm$8, 0)

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = 1

Vertices of the hyperbola = ($\pm$a, 0) = ($\pm$6, 0) $\Rightarrow$ a= 6



The foci = (±8, 0) = (±ae, 0)

⇒ ae = 8 [e is the eccentricity]

⇒ 6e = 8 [As a = 6]

⇒ e = $\frac{8}{6} = \frac{4}{3}$

We know that, e = $\sqrt{1+\frac{b^2}{a^2}}$

⇒ $\sqrt{1+\frac{b^2}{a^2}} = \frac{4}{3}$

⇒ $1+\frac{b^2}{a^2} = \frac{16}{9}$ [Squaring both sides]

⇒ $\frac{b^2}{a^2} = \frac{16}{9} - 1 = \frac{7}{9}$

⇒ $b^2 = \frac{7}{9}(a^2)$

⇒ $b^2 = 36 \times \frac{7}{9} = 4 \times 7 = 28$ [As a = 6]

So, the equation of the hyperbola is,

⇒ $\frac{x^2}{36} - \frac{y^2}{28} = 1$

OR

We have, foci of ellipse at (± 3, 0) which are on X-axis.

Therefore, equation of the ellipse is of the form

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

Its foci are (± ae, 0) = (± 3, 0)

∴ ae = 3

Now, $b^2 = a^2(1 - e^2)$

⇒ $a^2 e^2 = a^2 - b^2$

⇒ $9 = a^2 - b^2$ [∵ ae = 3] ...(ii)

Since, Eq. (i) passes through (4, 1)

∴ $\frac{16}{a^2} + \frac{1}{b^2} = 1$

⇒ $\frac{16}{9+b^2} + \frac{1}{b^2} = 1$ [putting Eq. (ii)]

⇒ $16b^2 + 9 + b^2 = b^2(9 + b^2)$

⇒ $17b^2 + 9 = 9b^2 + b^4$

⇒ $b^4 - 8b^2 - 9 = 0$

⇒ $(b^2 - 9)(b^2 + 1) = 0$

⇒ $b^2 = 9, -1$

But $b^2 \neq -1$

∴ $b^2 = 9$

From Eq. (ii), we get

$a^2 = 9 + b^2$

⇒ $a^2 = 9 + 9$

⇒ $a^2 = 18$

On putting the values of a^2 and b^2 in Eq. (i), we get

$\frac{x^2}{18} + \frac{y^2}{9} = 1$

$\Rightarrow x^2 + 2y^2 = 18$

This is the required equation of the ellipse

34. We have, $\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4}$ ... (i)

and $\frac{7x-1}{3} - \frac{7x+2}{6} > x$ ... (ii)

From inequality (i), we get

$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \Rightarrow \frac{16x-27}{12} < \frac{4x+3}{4}$

$\Rightarrow$ 16x - 27 < 12x + 9 [multiplying both sides by 12]

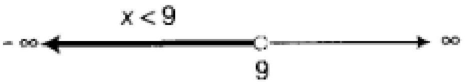
$\Rightarrow$ 16x - 27 + 27 < 12x + 9 + 27 [adding 27 on both sides]

$\Rightarrow$ 16x < 12x +36

$\Rightarrow$ 16x - 12x < 12x + 36 - 12x [subtracting 12x from bot sides]

$\Rightarrow$ 4x < 36 $\Rightarrow$ x < 9 [dividing both sides by 4]

Thus, any value of x less than 9 satisfies the inequality. So, the solution of inequality (i) is given by $x \in (-\infty, 9)$



From inequality (ii) we get,

$\frac{7x-1}{3} - \frac{7x+2}{6} > x \Rightarrow \frac{14x-2-7x-2}{6} > x$

$\Rightarrow$ 7x - 4 > 6x [multiplying by 6 on both sides]

$\Rightarrow$ 7x - 4 + 4 > 6x + 4 [adding 4 on both sides]

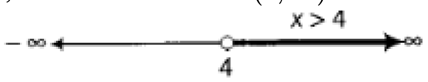
$\Rightarrow$ 7x > 6x + 4

$\Rightarrow$ 7x - 6x > 6x + 4 - 6x [subtracting 6x from both sides]

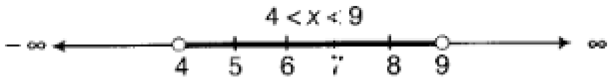
$\therefore$ x > 4

Thus, any value of x greater than 4 satisfies the inequality.

So, the solution set is $x \in (4, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



Clearly, the common value of x lie between 4 and 9.

Hence, the solution of the given system is, 4 < x < 9 i.e., $x \in (4, 9)$

35. We have to prove that $\cos^3 x \sin 3x + \sin^3 x \cos 3x = \frac{3}{4} \sin 4x$.

We know that,

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow 4 \cos^3\theta = \cos 3\theta + 3\cos\theta$

$\Rightarrow \cos^3\theta = \frac{\cos 3\theta + 3\cos\theta}{4}$ ... (i)

And similarly

$\Rightarrow \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\Rightarrow 4 \sin^3\theta = 3\sin\theta - \sin 3\theta$

$\Rightarrow \sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$ ... (ii)

Now,

LHS = $\cos^3 x \sin 3x + \sin^3 x \cos 3x$

Substituting the values from equation (i) and (ii), we get

$\Rightarrow \left(\frac{\cos 3x + 3\cos x}{4}\right) \sin 3x + \left(\frac{\cos 3x - 3\cos x}{4}\right) \cos 3x$

$= \frac{1}{4}$(sin 3x cos 3x + 3 sin 3x cosx + 3 sin x cos 3x - sin 3x cos 3x)

$= \frac{1}{4}$[3(sin 3x cos x + sin x cos 3x) + 0]

$= \frac{1}{4}$(3 sin(3x + x))

(as sin(x+y) = sin x cos y+cos x sin y)

$\Rightarrow \frac{3}{4}$ sin 4x

LHS = RHS

Hence Proved

OR

We have to prove that $\sin 6^o \sin 42^o \sin 66^o \sin 78^o = \frac{1}{16}$.

LHS = $\sin 6^o \sin 42^o \sin 66^o \sin 78^o$

By regrouping the LHS and multiplying and dividing by 4 we get,

$= \frac{1}{4}(2 \sin 66^o \sin 6^o)(2 \sin 78^o \sin 42^o)$

But 2 sin A sin B = cos (A - B) - cos (A + B)

Then the above equation becomes,

$=\frac{1}{4}(\cos(66^o - 6^o) - \cos(66^o + 6^o)) (\cos(78^o - 42^o) - \cos(78^o + 42^o))$

$=\frac{1}{4}(\cos(60^o) - \cos(72^o)) (\cos(36^o) - \cos(120^o))$

$=\frac{1}{4}(\cos(60^o) - \cos(90^o - 18^o)) (\cos(36^o) - \cos(180^o - 120^o))$

But $\cos(90^o - \theta) = \sin \theta$ and $\cos(180^o - \theta) = -\cos(\theta)$.

Then the above equation becomes,

$= \frac{1}{4}(\cos(60^o) - \cos(18^o)) (\cos(36^o) + \cos(60^o))$

Now, $\cos(36^o) = \frac{\sqrt{5}+1}{4}$

$\sin(18^o) = \frac{\sqrt{5}-1}{4}$

$\cos(60^o) = \frac{1}{2}$

Substituting the corresponding values, we get

$=\frac{1}{4}\left(\frac{1}{2} - \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)$

$=\frac{1}{4}\left(\frac{2-\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)$

$=\frac{1}{4}\left(\frac{3-\sqrt{5}}{4}\right)\left(\frac{3+\sqrt{5}}{4}\right)$

$=\frac{1}{4}\left(\frac{3^2-(\sqrt{5})^2}{4\times4}\right)$

$=\frac{1}{4}\left(\frac{9-5}{16}\right)$

$=\frac{1}{16}$

LHS = RHS

Hence proved.

Section E

36. i. The perimeter of 7th triangle

Perimeter of 1st triangle = $24 \times 3 = 72$ cm

Perimeter of 2nd triangle = $12 \times 3 = 36$ cm

Perimeter of 3rd triangle = $6 \times 3 = 18$ cm

Which is GP a= 72 and r= common ratio= $\frac{1}{2}$

a = 72, r = $\frac{1}{2}$

$S_n = \frac{a}{1-r}$ for infinite terms

$\Rightarrow S_n = \frac{72}{\frac{1}{2}} = 144$ cm

The sum of perimeter of all triangles = 144 cm

ii. Area of triangle 1st triangle = $\frac{\sqrt{3}}{4} \times 576$

Area of triangle 2nd triangle = $\frac{\sqrt{3}}{4} \times 144$

Which is in GP

$a = \frac{\sqrt{3}}{4} \times 576$ and $r = \frac{1}{4}$

Sum of areas of all triangles = $S_n = \frac{a}{1-r}$

$\Rightarrow S_n = \frac{\frac{\sqrt{3}}{4}\times576}{1-\frac{1}{4}} = \frac{\frac{\sqrt{3}}{4}\times576}{\frac{3}{4}}$

$\Rightarrow S_n = 192\sqrt{3}\ cm^2$

Sum areas of all triangles = $192\sqrt{3}$ sq cm

iii. a = 72, r = $\frac{1}{2}$, n = 6

$S_n = \frac{a(1-r^n)}{1-r} = \frac{72\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}}$

$\Rightarrow S_n = \frac{72\times63\times2}{64} = \frac{567}{4}$ cm

OR

Area of triangle 1st triangle = $\frac{\sqrt{3}}{4} \times 576$

Area of triangle 2nd triangle = $\frac{\sqrt{3}}{4} \times 144$

Which is in GP

a = $\frac{\sqrt{3}}{4} \times 576$ and r = $\frac{1}{4}$

Sum of areas of 4 triangles = $S_4 = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S_n = \frac{\frac{\sqrt{3}}{4}\times576\left(1-\left(\frac{1}{4}\right)^4\right)}{1-\frac{1}{4}}$

$\Rightarrow S_n = \frac{\frac{\sqrt{3}}{4}\times576\times255}{\frac{3}{4}\times256}$

$\Rightarrow S_n = \frac{765\sqrt{3}}{4}$ cm^2

37. i. H: Student read Hindi newspaper

E: Student read English newspaper

P(H) = $\frac{60}{100}$, P(E) = $\frac{40}{100}$, $P(H\cap E) = \frac{20}{100}$

$P(E \cup H) = P(E) + P(H) - P(E \cap H)$

$\Rightarrow P(E\cup H) = \frac{40}{100} + \frac{60}{100} - \frac{20}{100} = \frac{80}{100}$

$\Rightarrow P(E\cup H) = \frac{4}{5}$ = 80%

$\Rightarrow$ 80% of students read English or Hindi newspaper.

ii. H: Student read Hindi newspaper

E: Student read English newspaper

P(H) = $\frac{60}{100}$, P(E) = $\frac{40}{100}$, $P(H\cap E) = \frac{20}{100}$

$P(E \cup H)' = 1 - P(E \cup H)$

$\Rightarrow P(E\cup H)' = 1 - \frac{4}{5} = \frac{1}{5}$ = 20%

$\Rightarrow$ 20% of students read neither English nor Hindi newspapers.

iii. H: Student read Hindi newspaper

E: Student read English newspaper

P(H) = $\frac{60}{100}$, P(E) = $\frac{40}{100}$, $P(H\cap E) = \frac{20}{100}$

$P(E' \cap H) = P(H) - P(E \cap H)$

$\Rightarrow P(E'\cap H) = \frac{60}{100} - \frac{20}{100} = \frac{40}{100}$ = 40%

$\Rightarrow$ 40% of students read only Hindi newspapers.

OR

H: Student read Hindi newspaper

E: Student read English newspaper

P(H) = $\frac{60}{100}$, P(E) = $\frac{40}{100}$, $P(H\cap E) = \frac{20}{100}$

$P(E \cap H') = P(E) - P(E \cap H)$

$\Rightarrow P(E'\cap H) = \frac{40}{100} - \frac{20}{100} = \frac{20}{100}$ = 20 %

$\Rightarrow$ 20 % of students read only English newspaper.

38. i. Atleast one = 11 + 9 + 5 + 4 - 2(3)

= 29 − 6 = 23

$\Rightarrow$ None = 25 − 23 = 2

ii. The number of students who reading atleast one of the subject is 23.

iii. Only maths = 15 - 9 - 5 + 3 = 4

Only physics = 12 - 9 - 4 + 3 = 2

Only chemistry = 5 $\Rightarrow$ Total = 11

OR

The number of students who reading only mathematics is 4.